Laboratori Nazionali di Frascati

LNF-63/45 (1963)

R. Gatto: SPIN-STATISTICS CONNECTION FOR STRANGE PARTICLES.

Estratto dal: Phys. Letters 5, 56 (1963).

meson theory. The crossed graphs lead to an attractive potential, as do also the uncrossed graphs; however, it is well known that the fourth-order potential vanishes identically for neutral scalar meson theory, these attractive potentials being cancelled by the "non-adiabatic corrections" to the first-order potential [14] (here, the first-order potential contributes only to $V(\Lambda\Lambda,\Sigma\Sigma)$, so these "non-adiabatic corrections" arise from the coupling between the Λ-Λ and Σ-Σ channels). The inclusion of the $\Sigma\Lambda$ mass difference and of recoil in the Σ-Σ channel will effectively suppress the repulsion due to these "non-adiabatic correction" terms, so that the net Λ-Λ potential may be expected to be attractive for $f_{\Sigma\Sigma} = 0$. In view of the importance of the coupling potential $V(\Lambda\Lambda,\Sigma\Sigma)$ here, the coupling $f_{\Sigma\Sigma}$ will actually be rather important, even for small $f_{\Sigma\Sigma} \approx -0.1$ to -0.2, since it will contribute to the fourth-order term of $V(\Lambda\Lambda,\Sigma\Sigma)$. For odd $\Sigma\Lambda$ parity, a quantitative estimate of the strength to be expected for the Λ-Λ interaction will require a detailed treatment of these effects.

In conclusion, we wish to acknowledge a valuable correspondence with Dr. J. J. De Swart, who pointed out an important error in the original manuscript.

This work was supported by the U.S. Atomic Energy Commission.

References

1) M.Danysz et al., Phys. Rev. Letters (1963), to be published.
2) R.H.Dalitz and B.W.Downs, Phys.Rev. 111 (1958) 967.
3) J.De Swart and C.Iddings, (a) Phys. Rev. 128 (1962) 2810; (b) Phys. Rev. 130 (1963) 319.
4) J.De Swart and C.Dullemond, Ann.Phys. (N.Y.) 19 (1962) 458.
5) B.W.Downs and R.H.Dalitz, Phys.Rev. 114 (1959) 593.
6) K.Suh, Phys. Rev. 111 (1958) 941.
7) Cf. R.H.Dalitz, Proc. Futherford Jubilee Int. Conf. (Heywood, London, 1962) p. 103.
8) A.Martin and K.C.Wali, Coupled channel approach to $J = \frac{3}{2}^+$ resonances in the unitary symmetry model, to be published (1963).
9) J.De Swart, private communication (1963).
10) A.Deloff, private communication from Professor M. Danysz (1963).
11) S.Iwao, Nuclear Phys. 26 (1961) 1.
12) J.Sakurai, Ann. Phys. (N.Y.) 11 (1960) 1.
13) M.Gell-Mann, Phys. Rev. 125 (1962) 1067.
14) E.g.: E.A.Power, Nuovo Cimento 12 (1954) 330.

* * * * *

LNF-63/45

SPIN-STATISTICS CONNECTION FOR STRANGE PARTICLES

R. GATTO

Istituto di Fisica dell'Università, Firenze
Laboratori Nazionali di Frascati del C.N.E.N., Frascati, Roma

Received 13 May 1963

It is the purpose of this note to point out the possible relevance of general experimental results on proton-antiproton annihilation into strange particles to the problem of the spin-statistics connection for strange particles.

According to Pauli [1] particles with integral spin obey Bose statistics and particles with half-integral spin obey Fermi statistics. Pauli's argument is based for integral spins on the assumption of vanishing commutator for space-like distances, and for half-integral spins on the assumption of a positive definite energy. Recent derivations have been given by Burgoyne [2] and by Lüders and Zumino [3]. The derivation presented by Lüders and Zumino is based on five general postulates: (i) invariance with respect to proper Lorentz group; (ii) vanishing commutators or anticommutators for space-like distances; (iii) vacuum as state of lowest energy; (iv) positive definite metric; and (v), the vacuum is not identically annihilated by a field operator.

If a spin-zero field $\theta(x)$

$$\theta(x) = \frac{1}{\sqrt{v}} \sum_{\boldsymbol{k}} \frac{1}{\sqrt{2k_0}} [a(\boldsymbol{k})\, e^{ikx} + b^+(\boldsymbol{k})\, e^{-ikx}] \tag{1}$$

is quantised with anticommutators

$$[a(\boldsymbol{k}), a^+(\boldsymbol{k}')]_+ = [b(\boldsymbol{k}), b^+(\boldsymbol{k}')]_+ = \delta_{\boldsymbol{k},\boldsymbol{k}'} \, , \tag{2}$$

one has

$$[\theta(x), \theta(x')]_+ = 0 \, , \quad [\theta(x), \theta^+(x')]_+ = \Delta_1(x-x') \, . \tag{3}$$

The Green function $\Delta_1(x)$

$$\Delta_1(x) = \frac{1}{(2\pi)^3} \int d^4k \, e^{-ikx} \, \delta(k^2+m^2) \tag{4}$$

does not vanish outside the light-cone. This fact suggests that, among the above postulates, the causality postulate ii (at least) would be violated. The Green function $\Delta_1(x)$ falls off exponentially for large space-like distances ($\gg m^{-1}$). One might thus hope that only microscopic causality, for distances $\lesssim m^{-1}$ is violated.

Quantisation of a spin $\frac{1}{2}$ field $\Lambda(x)$ with commutators leads to

$$[\Lambda(x), \Lambda(x')] = 0 \ , \quad [\Lambda(x), \bar{\Lambda}(x')] = -\,\mathrm{i}\, S(x-x') \ , \tag{5}$$

where the Green function $S(x)$ vanishes outside the light-cone. The causality postulate (ii) does not seem to be necessarily violated. In such a case, however, some of the other postulates must be violated (for instance postulate iii).

At the Kiev Conference in 1959 Drell presented speculations on the possibility that K particles obey Fermi statistics and Λ particles obey Bose statistics [4]. By explicit evaluation of the S matrix up to fourth-order he showed the possibility that such a theory be consistent.

In order to draw verifiable conclusions on physical processes we shall here assume, per absurdum, that a theory with K-mesons obeying anticommutation rules is consistent and admits of interpretation in terms of asymptotic states. Furthermore we assume that parity and particle-antiparticle conjugation are conserved for all strong interactions including strange particles. Specifically, following Lüders [5] we assume that one can define ingoing and outgoing parities P_{in} and P_{out} such that, in particular,

$$P_{\mathrm{out}}\theta_{\mathrm{out}}(\boldsymbol{x}, x_0)P_{\mathrm{out}}^{-1} = \epsilon_P\theta_{\mathrm{out}}(-\boldsymbol{x}, x_0) \tag{6}$$

with $\epsilon_P = \pm 1$ and for which

$$P_{\mathrm{in}} = P_{\mathrm{out}} \ . \tag{7}$$

Similarly we assume that one can define C_{in} and C_{out}, such that in particular

$$C_{\mathrm{out}}\theta^{+}_{\mathrm{out}}(x)C^{-1}_{\mathrm{out}} = \epsilon_C\theta_{\mathrm{out}}(x) \tag{8}$$

with $|\epsilon_C|^2 = 1$ and for which

$$C_{\mathrm{in}} = C_{\mathrm{out}} \ . \tag{9}$$

Let us now consider an outgoing state of a K and a $\overline{\mathrm{K}}$ with angular momentum J, M. It is represented by

$$\int \mathrm{d}\omega_{\boldsymbol{k}}\, Y_J^M(\boldsymbol{k})\, a^{+}_{\mathrm{out}}(\boldsymbol{k})\, b^{+}_{\mathrm{out}}(-\boldsymbol{k})|0\rangle \ , \tag{10}$$

where $\mathrm{d}\omega_{\boldsymbol{k}}$ is the element of solid angle, $Y_J{}^M$ is a spherical harmonics and $\boldsymbol{k}$ is the c.m. momentum of the K particle. From (1), applied to the K field, (6), and (10), one has that the parity P_{out} is for the state (10) $P_{\mathrm{out}} = (-1)^J$, independently of the commutation relations of the K field. By applying C_{out} to the state (10) one obtains, from (6) and (8)

$$(-1)^J \int \mathrm{d}\omega_{\boldsymbol{k}}\, Y_J^M(\boldsymbol{k})\, b^{+}_{\mathrm{out}}(-\boldsymbol{k})\, a^{+}_{\mathrm{out}}(\boldsymbol{k})|0\rangle \ . \tag{11}$$

If now a^+ and b^+ anticommute one has that the particle-antiparticle conjugation C_{out} is, for the state (10), $C_{\mathrm{out}} = (-1)^{J+1}$. From these conclusions it follows that the reaction

$$\mathrm{p} + \bar{\mathrm{p}} \rightarrow \mathrm{K} + \overline{\mathrm{K}} \ , \tag{12}$$

occurring both at rest or in flight, is forbidden. In fact, for annihilation from singlet states of the p - $\bar{\mathrm{p}}$ system $P_{\mathrm{in}} = (-)^{J+1}$ and $C_{\mathrm{in}} = (-)^J$. Therefore both (7) and (9) cannot be satisfied. For annihilation from triplet states one has in general $P_{\mathrm{in}} = C_{\mathrm{in}}$ so that (7) and (9) cannot be simultaneasly verified.

The evidence for the annihilation mode (12) has by now become quite convincing [6]. We are thus led to the conclusion that the hypothesis of anticommutation relations for the K field is in conflict with experimental data, at least as long as the formalism admits of the usual interpretation in terms of asymptotic states on which the symmetry operations can be defined as for the ordinary theory.

The assumption of commutation relations for the Λ field does not lead (if one follows a reasoning analogous to the one used above) to the complete forbiddenness of the annihilation process

$$\mathrm{p} + \bar{\mathrm{p}} \rightarrow \Lambda + \bar{\Lambda} \ . \tag{13}$$

With commutation relations for the Λ field, reaction (13) would occur only via triplet-singlet and singlet-triplet transitions for those triplet states for which $J = L$ (J = total angular momentum, L = orbital angular momentum). In particular (13) would be forbidden from S-states. Such consequences could be easily checked experimentally.

We do not comment here on the question whether strange particles obey "parastatistics" in which the commutation or anticommutation rules are assumed to be trilinear, or quadrilinear, etc., and on which much interest has recently arisen [7].

References

1) W. Pauli, Phys. Rev. 58 (1940) 716.
2) N. Burgoyne, Nuovo Cimento 8 (1958) 607.
3) G. Lüders and B. Zumino, Phys. Rev. 110 (1958) 1450.
4) S. Drell, Proc. of the int. conf. on high energy physics, Kiev 1960, edited by the Academy of Sciences of the USSR, p. 137.
5) G. Lüders, Rendiconti della scuola "Enrico Fermi", Corso XI, Interazioni deboli, Zanichelli - Bologna 1959.
6) E.g.: Armenteros et al., Proc. of the 1962 Geneva conf. on elementary particles, Geneva 1962, p. 351.
7) H. Feshbach, Physics Letters 3 (1962/63) 317.

* * * * *